Local stabilization of networked linear systems with decentralized switching controllers

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Summary

Switched affine systems

Consider a switched affine system

$$\dot{x} = A_{\sigma(x)}x + b_{\sigma(x)}$$

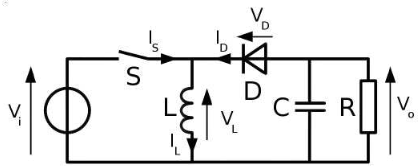
$x \in \mathbb{R}^n$ is the system state

$A_i \in \mathbb{R}^{n\times n}$, $b_i \in \mathbb{R}^n$ are known for $i \in \mathcal{I}_N = \{1, \cdots, N\}$.

State dependent switching control law

$$\sigma = \sigma(x), \sigma : \mathbb{R}^n \to \mathcal{I}_N$$

Motivational example: buck boost converter



Buck boost converter example

$$\dot{x} = \begin{cases} \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix} x + \begin{bmatrix} \frac{V_i}{L} \\ 0 \end{bmatrix}, & \text{if S is ON} \\ \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{RC} \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \end{bmatrix}, & \text{if S is OFF} \end{cases}$$

- $x = \begin{bmatrix} I_L & V_0 \end{bmatrix}^T$
- When the supply voltage is constant, the model is a switched affine system

$$\dot{x} = A_\sigma x + b_\sigma, \qquad \sigma \in \{1, 2\}.$$

- The switching control law : setting S to ON or OFF.

Challenges

$$\dot{x} = A_{\sigma(x)}x + b_{\sigma(x)}, \qquad x_0 \in \mathbb{R}^n, \tag{1}$$

- State dependent switching laws:

$$\sigma : \mathbb{R}^n \longrightarrow \{1, \cdots, N\}.$$

Challenges and difficulties:

- Problem of the equilibrium
- Stabilization to a desired reference point by switching

Existence of a common quadratic Lyapunov function[1]

If there exist $\alpha_i \geq 0$ for all $i \in \{1, \cdots, N\}$ such that $\sum_{i=1}^{N} \alpha_i = 1$ and

$$\sum_{i=1}^{N} \alpha_i b_i = 0.$$

and there exist a matrix

$$P \succ 0$$

such that

$$A_i^T P + P A_i \prec 0, \qquad \forall i \in \{1, \cdots, N\},$$

then the switching law

$$\sigma(x) \in \arg \min_{i \in \{1, \cdots, N\}} x^T P b_i$$

ensures that the equilibrium $x_e = 0$ of the system (1) globally asymptotically stable.

[1] G Silva Deaecto et al. "Switched affine systems control design with application to DC–DC converters". In: *IET control theory & applications* 4.7 (2010), pp. 1201–1210.

State of the art

- For all other equilibrium satisfying

$$\sum_{i=1}^{N} \alpha_i (A_i x^* + b_i) = 0, \qquad \alpha_i \geq 0, \qquad \sum_{i=1}^{N} \alpha_i = 1,$$

- Stabilization of the origin requires the existence of α such that

$$\sum_{i=1}^{N} \alpha_i b_i = 0, \qquad \alpha_i \geq 0, \qquad \sum_{i=1}^{N} \alpha_i = 1,$$

- The following transformation is considered

$$\xi = x - x^*.$$

- Stabilization of the equilibrium x^* can be transformed to the stabilization of the origin $x_e = 0$

State of the art

Existence of a Hurwitz convex combination[2]

If there exist $\alpha_i \geq 0$, $i \in \{1, \cdots, N\}$ such that $\sum_{i=1}^{N} \alpha_i = 1$ and

$$b_{eq} = 0, \qquad \text{where} \qquad b_{eq} = \sum_{i=1}^{N} \alpha_i b_i,$$

and

$$A_{eq} \quad \text{is Hurwitz, i.e there exists} \quad P \succ 0, \quad \text{such that} \quad A_{eq}^T P + P A_{eq} \prec 0, \quad \text{where}$$

$$A_{eq} = \sum_{i=1}^{N} \alpha_i A_i$$

then the switching law

$$\sigma(x) \in \arg \min_{i \in \{1, \cdots, N\}} \left\{ x^T P (A_i x + b_i) \right\}. \tag{2}$$

stabilizes asymptotically the origin $x_e = 0$ of the system (1).

[2] Paolo Bolzern and William Spinelli. “Quadratic stabilization of a switched affine system about a nonequilibrium point”. In: *Proceedings of the 2004 American Control Conference*. Vol. 5. IEEE. 2004, pp. 3890–3895.

Local stabilization based on the existence of a continuous controller[3]

The system description (1) is equivalent to a bilinear system of the form

$$\dot{x} = Ax + \sum_{i=1}^{m} N_i x u(i) + Bu, \qquad m = N-1, \qquad , u \in \{v_1, \cdots, v_N\}$$

where A, $N_1, \cdots, N_m$ and B are matrices to be determined.

If there exist $\alpha_i \geq 0$ for all $i \in \{1, \cdots, N\}$ such that $\sum_{i=1}^{N} \alpha_i = 1$ and $\sum_{i=1}^{N} \alpha_i b_i = 0$. and a continuous controller $k^c(x)$, with $k^c(0) = 0$, that locally stabilizes system (7), then the switching law

$$\sigma(x) \in \arg \min_{i \in \{1, \cdots, N\}} \frac{\partial V(x)}{\partial x}(A_i x + b_i),$$

ensures that the origin of the switched system (1) is locally asymptotically stable.

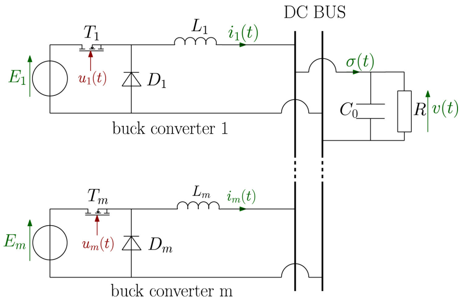
($V(x)$ is a function defined in a neighborhood of the origin and always exists if the continuous controller exists).

[3] Laurentiu Hetel and Emmanuel Bernuau. "Local stabilization of switched affine systems". In: *IEEE Transactions on Automatic Control* 60.4 (2014), pp. 1158–1163.

Problem statement

In practice:

- Very large switching systems
- Networked systems
- Centralized controllers [4]



Interconnection of buck converters

[4] Aboubacar Ndoye et al. "Switching control design for LTI system with uncertain equilibrium: Application to parallel interconnection of DC/DC converters". In: *Automatica* 145 (2022), p. 110522

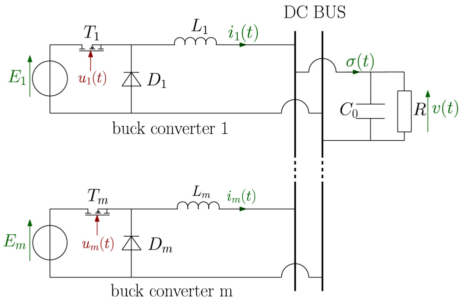
Problem statement

In practice:

- Very large switching systems
- Networked systems
- Centralized controllers [4]

In this work we consider:

- Switching control for networks of switched affine systems
- Decentralized controllers
- Scalable design conditions



Interconnection of buck converters

[4] Ndoye et al., "Switching control design for LTI system with uncertain equilibrium: Application to parallel interconnection of DC/DC converters"

Problem statement

$$\dot{x}_i = \bar{A}x_i + \bar{b}_{\sigma_i(x_i)} + \sum_{j=1}^{n} a_{ij}^M F x_j, \qquad x_i(0) \in \mathbb{R}^{n_x}, \qquad \forall i \in \{1, \cdots, n\}, \tag{3}$$

- $\bar{A}$, $\bar{b}_1, \cdots, \bar{b}_N$, F and $A^M = [a_{ij}^M]_{i\in\{1,\cdots,n\}, j\in\{1,\cdots,n\}}$
- n: number of subsystems on the network
- N: number of possible modes for every subsystem
- Switching control laws

$$\sigma_i(x_i) : \mathbb{R}^{n_x} \to \{1, \cdots, N\}, \tag{4}$$

Problem statement

$$\dot{x}_i = Ax_i + Bu_i + \sum_{j=1}^{n} a_{ij}^M F x_j, \qquad x_i(0) \in \mathbb{R}^{n_x}, \qquad \forall i \in \{1, \cdots, n\}, \tag{5}$$

$$u_i \in \mathcal{V}, \qquad \mathcal{V} = \{v^1, \cdots, v^N\} \subset \mathbb{R}^{n_u},$$

Assumptions

- $\mathcal{V}$ is non-empty and contains 0 in the interior of its convex hull (example $\{-1, 1\}$)
- A^M is symmetric

System (5) can be seen as a network of switched systems:

$$\bar{A} = A, \qquad \bar{b}_{\sigma(x_i)} = Bu_i, \qquad u_i \in \mathcal{V}, \qquad \forall i \in \{1, \cdots, n\},$$

$$\dot{x}_i = \bar{A}x_i + \bar{b}_{\sigma_i(x_i)} + \sum_{j=1}^{n} a_{ij}^M F x_j, \qquad \forall i \in \{1, \cdots, n\}$$

Problem statement

$$\dot{x}_i = Ax_i + Bu_i + \sum_{j=1}^{n} a_{ij}^M F x_j, \qquad x_i(0) \in \mathbb{R}^{n_x}, \qquad \forall i \in \{1, \cdots, n\}, \tag{5}$$
$$u_i \in \mathcal{V}, \qquad \mathcal{V} = \{v^1, \cdots, v^N\} \subset \mathbb{R}^{n_u},$$

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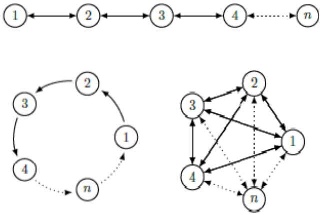


Different possible structures

Problem statement: decentralized control

$$\dot{x}_i = Ax_i + Bu_i + \sum_{j=1}^{n} a_{ij}^M F x_j, \qquad x_i(0) \in \mathbb{R}^{n_x}, \qquad \forall i \in \{1, \cdots, n\}, \tag{5}$$

$$u_i \in \mathcal{V}, \qquad \mathcal{V} = \{v^1, \cdots, v^N\} \subset \mathbb{R}^{n_u},$$

Assumptions

- $\mathcal{V}$ is non-empty and contains 0 in the interior of its convex hull
- A^M is symmetric

Objective: finding conditions such that the switching control laws

$$u_i = \kappa_i(x_i), \qquad \kappa_i : \mathbb{R}^{n_x} \to \mathcal{V}, \qquad i \in \{1, \cdots, n\}$$

guarantee that the origin of the networked system (5) is locally exponentially stable

Scalable design conditions

Theorem

Consider the system (5), with connectivity matrix A^M whose eigenvalues are $\lambda_1, \cdots, \lambda_n$. Let there exist $P = P^T \in \mathbb{R}^{n_x \times n_x} \succ 0$, $K \in \mathbb{R}^{n_u \times n_x}$ and $\delta > 0$ such that

$$(A + \lambda F + BK)^T P + P(A + \lambda F + BK) < -2\delta P, \qquad \forall \lambda \in \{\lambda_{min}, \lambda_{max}\}.$$

Define: $C_v(K) = \{z \in \mathbb{R}^{n_x} : Kz \in \mathrm{conv}(\mathcal{V})\}$. Consider an ellipsoid $\mathcal{E}(P, \gamma)$, such that $\mathcal{E}(P, \gamma) \subset C_v(K)$. Denote, $\bar{P} = I_n \otimes P$. Consider the set of control laws

$$u_i = \kappa_i(x_i) \in \arg\min_{v \in \mathcal{V}} x_i^T PBv, \tag{6}$$

for every subsystem $i \in \{1, \cdots, n\}$. Then, the origin of the networked system (5),(6) is locally exponentially stable, and $\mathcal{E}(\bar{P}, \gamma)$ is an estimate of its domain of attraction.

Idea of the proof

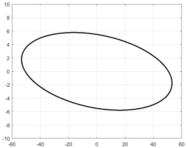
- Closed loop system

$$\dot{x} = \bar{A}x + (A^M \otimes F)x + \bar{B}u$$

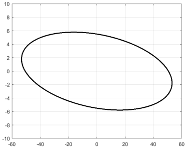
$$\bar{A} = I_n \otimes A,\ \bar{B} = I_n \otimes B\ ,\ x = \begin{bmatrix} x_1^T & \cdots & x_n^T \end{bmatrix}^T\ ,\ u = \begin{bmatrix} u_1^T & \cdots & u_n^T \end{bmatrix}^T \text{ where}$$

$$u_i = \kappa_i(x_i) \in \arg\min_{\nu \in \mathcal{V}} x_i^T P B \nu$$

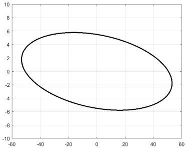
- We look for the existence of a Lyapunov function in the ellipsoid $\mathcal{E}(\bar{P}, \gamma)$.



$x_1 \in \mathcal{E}(P, \frac{\gamma}{n})$



$x_2 \in \mathcal{E}(P, \frac{\gamma}{n})$



$x_3 \in \mathcal{E}(P, \frac{\gamma}{n})$

$$x = \begin{bmatrix} x_1^T & x_2^T & x_3^T \end{bmatrix}^T$$

$$x \in \mathcal{E}(\bar{P}, \gamma)$$

$\gamma_{\bar{n}}$

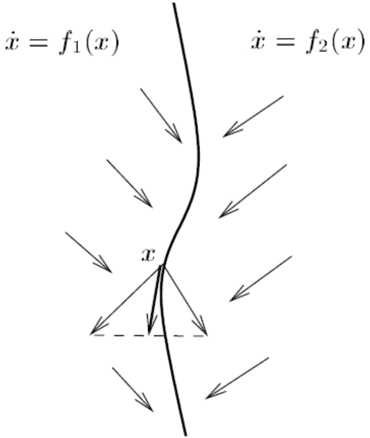
Filippov solutions for discontinuous dynamical systems

- Set-valued map
- Differential inclusion

$$\dot{x} \in F(x)$$

- Lyapunov condition:

$$\sup_{y \in \mathcal{F}(x)} \frac{\partial w(x)}{\partial x} y \leq -2\delta w(x),$$



vector field of a discontinuous dynamical system

Steps of the proof

1. Stability of the block-diagonal system guaranteed by $\bar{K}z = (I_n \otimes K)z$

$$\frac{\partial w(z)}{\partial z}(\bar{A} + \Lambda \otimes F + \bar{B}\bar{K})z \leq -2\delta w(z), \qquad \forall z \in \mathbb{R}^{n.n_x}. \tag{7}$$

2. Stability of the networked system guaranteed by the continuous control $\bar{K}x$

$$\frac{\partial w(x)}{\partial x}\left(\bar{A} + (A^M \otimes F) + \bar{B}\bar{K}\right)x \leq -2\delta w(x), \qquad \forall x \in \mathbb{R}^{n.n_x}. \tag{8}$$

3. Local stability guaranteed bu the switching control

$$\frac{\partial w(x)}{\partial x}\left(\bar{A}x + (A^M \otimes F)x + \bar{B}\bar{v}^*\right) \leq -2\delta w(x), \qquad x \in \mathcal{E}(\bar{P}, \gamma) \tag{9}$$

with

$$\bar{v}^* = \left[(v_1^*)^T \quad \cdots \quad (v_n^*)^T\right]^T \in \mathbb{R}^{n.n_u}, \qquad v_i^* \in \arg\min_{v \in \mathcal{V}} x_i^T PBv,$$

Steps of the proof

1.
$$\frac{\partial w(z)}{\partial z}(\bar{A} + \Lambda \otimes F + \bar{B}\bar{K})z \leq -2\delta w(z), \qquad \forall z \in \mathbb{R}^{n.n_x}. \tag{7}$$

2.
$$\frac{\partial w(x)}{\partial x}(\bar{A} + (A^M \otimes F) + \bar{B}\bar{K})x \leq -2\delta w(x), \qquad \forall x \in \mathbb{R}^{n.n_x}. \tag{8}$$

3.
$$\frac{\partial w(x)}{\partial x}(\bar{A}x + (A^M \otimes F)x + \bar{B}\bar{v}^*) \leq -2\delta w(x), \qquad x \in \mathcal{E}(\bar{P}, \gamma) \tag{9}$$

with
$$\bar{v}^* = \begin{bmatrix} (v_1^*)^T & \cdots & (v_n^*)^T \end{bmatrix}^T \in \mathbb{R}^{n.n_u}, \qquad v_i^* \in \arg\min_{v \in \mathcal{V}} x_i^T PBv,$$

1. Scalable conditions:

$$(A + \lambda_i F + BK)^T P + P(A + \lambda_i F + BK) < -2\delta P, \qquad \forall i \in \{1, \cdots, n\}. \tag{10}$$

$$\Lambda = \mathrm{diag}(\lambda_1, \cdots, \lambda_n), \qquad w : \mathbb{R}^{nn_x} \to \mathbb{R}, w(z) := z^T \bar{P} z, \qquad \bar{M} = I_n \otimes M = \begin{bmatrix} M & 0 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & M \end{bmatrix}$$

Steps of the proof

1. $$\frac{\partial w(z)}{\partial z}(\bar{A} + \Lambda \otimes F + \bar{B}\bar{K})z \leq -2\delta w(z), \qquad \forall z \in \mathbb{R}^{n.n_x}. \tag{7}$$

2. $$\frac{\partial w(x)}{\partial x}(\bar{A} + (A^M \otimes F) + \bar{B}\bar{K})x \leq -2\delta w(x), \qquad \forall x \in \mathbb{R}^{n.n_x}. \tag{8}$$

3. $$\frac{\partial w(x)}{\partial x}(\bar{A}x + (A^M \otimes F)x + \bar{B}\bar{v}^*) \leq -2\delta w(x), \qquad x \in \mathcal{E}(\bar{P}, \gamma) \tag{9}$$

   with

   $$\bar{v}^* = \left[(v_1^*)^T \quad \cdots \quad (v_n^*)^T\right]^T \in \mathbb{R}^{n.n_u}, \qquad v_i^* \in \arg\min_{v \in \mathcal{V}} x_i^T PBv,$$

2. Change of coordinates $x = (T^T \otimes I)z$ where $TA^M T^T = \Lambda$.

Steps of the proof

1. $$\frac{\partial w(z)}{\partial z}(\bar{A} + \Lambda \otimes F + \bar{B}\bar{K})z \leq -2\delta w(z), \qquad \forall z \in \mathbb{R}^{n.n_x}. \tag{7}$$

2. $$\frac{\partial w(x)}{\partial x}(\bar{A} + (A^M \otimes F) + \bar{B}\bar{K})x \leq -2\delta w(x), \qquad \forall x \in \mathbb{R}^{n.n_x}. \tag{8}$$

3. $$\frac{\partial w(x)}{\partial x}\left(\bar{A}x + (A^M \otimes F)x + \bar{B}\bar{v}^*\right) \leq -2\delta w(x), \qquad x \in \mathcal{E}(\bar{P}, \gamma) \tag{9}$$

   with

   $$\bar{v}^* = \left[(v_1^*)^T \quad \cdots \quad (v_n^*)^T\right]^T \in \mathbb{R}^{n.n_u}, \qquad v_i^* \in \arg\min_{v \in \mathcal{V}} x_i^T PBv,$$

3. $$x \in \mathcal{E}(\bar{P}, \gamma) \implies x_i \in \mathcal{E}(P, \gamma),\ \forall i \in \{1, \cdots, n\} \implies Kx_i \in \mathrm{conv}(\mathcal{V}),\ \forall i \in \{1, \cdots, n\}$$

   $$\implies x_i^T PBv_i^* \leq x_i^T PBKx_i, \qquad \forall i \in \{1, \cdots, n\},$$

Steps of the proof

- Possible control

$$\mathcal{U}^*(x) = \left\{ v \in \mathbb{R}^{nn_u}, v = \left[(v_1^*)^T \quad \cdots \quad (v_n^*)^T \right]^T : \; v_i^* \in \arg\min_{\nu \in \mathcal{V}} x_i^T P B \nu, \; \forall i \in \{1, \cdots, n\} \right\}$$

- Differential inclusion

$$\dot{x} \in \mathcal{F}(x),$$
$$\mathcal{F}(x) = \underset{u \in \mathcal{U}^*(x)}{\mathrm{conv}} \left\{ \bar{A}x + (A^M \otimes F)x + \bar{B}u \right\},$$

- The Lyapunov condition for the discontinuous system

$$\sup_{y \in \mathcal{F}(x)} \frac{\partial w(x)}{\partial x} y \leq -2\delta w(x), \qquad \forall x \in \mathcal{E}(\bar{P}, \gamma), \tag{10}$$

for the function $w(x) = x^T \bar{P} x$.

Further discussion about the result

LMIs for control design

For fixed R and δ, solve with respect to Q, μ and ξ

$$
\begin{aligned}
&Q(A+\lambda F)^T + (A+\lambda F)Q - \mu BB^T + 2\delta Q \prec 0, \quad && \forall \lambda \in \{\lambda_{min}, \lambda_{max}\}, \\
&\begin{bmatrix} Q & -0.5\mu Bh_j \\ (-0.5\mu Bh_j)^T & 1 \end{bmatrix} \succ 0, && \forall j \in \{1, \cdots, n_h\}, \\
&\begin{bmatrix} \xi R & I \\ I & Q \end{bmatrix} \succ 0.
\end{aligned}
$$

For fixed R and δ, solve with respect to Q, μ and ξ

$$\begin{aligned}
&Q(A+\lambda F)^T + (A+\lambda F)Q - \mu BB^T + 2\delta Q \prec 0, \quad \forall \lambda \in \{\lambda_{min}, \lambda_{max}\},\\
&\begin{bmatrix} Q & -0.5\mu Bh_j \\ (-0.5\mu Bh_j)^T & 1 \end{bmatrix} \succ 0, \quad \forall j \in \{1, \cdots, n_h\},\\
&\begin{bmatrix} \xi R & I \\ I & Q \end{bmatrix} \succ 0.
\end{aligned}$$

1.

$$\lambda_{min} = \min(\lambda_1, \cdots, \lambda_n) \qquad \lambda_{max} = \max(\lambda_1, \cdots, \lambda_n)$$

By taking

$$P = Q^{-1}, \qquad K = -\frac{\mu}{2} B^T P$$

we can easily verify that

$$(A + \lambda_i F + BK)^T P + P(A + \lambda_i F + BK) < -2\delta P, \qquad \forall i \in \{1, \cdots, n\}$$

For fixed R and δ, solve with respect to Q, μ and ξ

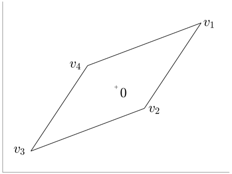
$$
\begin{aligned}
&Q(A+\lambda F)^T + (A+\lambda F)Q - \mu BB^T + 2\delta Q \prec 0, \quad \forall \lambda \in \{\lambda_{min}, \lambda_{max}\},\\
&\begin{bmatrix} Q & -0.5\mu Bh_j \\ (-0.5\mu Bh_j)^T & 1 \end{bmatrix} \succ 0, \quad \forall j \in \{1, \cdots, n_h\},\\
&\begin{bmatrix} \xi R & I \\ I & Q \end{bmatrix} \succ 0.
\end{aligned}
$$

2.

$$
\begin{aligned}
conv(\mathcal{V}) &= conv(v_1, \cdots, v_N)\\
&= \left\{ y \in \mathbb{R}^m : \; h_i^T y \le 1, \; \forall i \in \{1, \cdots, n_h\} \right\}
\end{aligned}
$$

$\Longrightarrow$

$$\mathcal{E}(P, \gamma) \subset C_v(K)$$



A polytope in $\mathbb{R}^2$

LMIs for control design

For fixed R and δ, solve with respect to Q, μ and ξ

$$Q(A+\lambda F)^T + (A+\lambda F)Q - \mu BB^T + 2\delta Q \prec 0, \quad \forall \lambda \in \{\lambda_{min}, \lambda_{max}\},$$

$$\begin{bmatrix} Q & -0.5\mu Bh_j \\ (-0.5\mu Bh_j)^T & 1 \end{bmatrix} \succ 0, \quad \forall j \in \{1, \cdots, n_h\},$$

$$\begin{bmatrix} \xi R & I \\ I & Q \end{bmatrix} \succ 0.$$

3.

$$\mathcal{E}(R, \frac{1}{\xi}) \subset \mathcal{E}(P, 1)$$

Numerical example

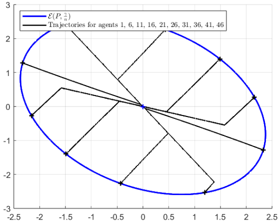
We consider a network of ***50*** subsystems with an all-to-all interconnection, whose dynamics are given by (5) with

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 1 \\ -1 & 0.5 \end{bmatrix}, \qquad F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\mathcal{V} = \left\{ \begin{bmatrix} 25 \\ 25 \end{bmatrix}, \begin{bmatrix} -25 \\ 25 \end{bmatrix}, \begin{bmatrix} 25 \\ -25 \end{bmatrix}, \begin{bmatrix} -25 \\ -25 \end{bmatrix} \right\}.$$

We choose $\delta = 0.5$, $\gamma = 1$ and $R = I$, the conditions of the Theorem are satisfied for

$$P = \begin{bmatrix} 0.0039 & 0.0011 \\ 0.0011 & 0.0033 \end{bmatrix}.$$



domain of attraction and trajectories for some subsystems for the given example.

Future works

- Extend conditions to switched affine systems (with switching state matrix)

$$\dot{x}_i = Ax_i + Bu_i + \sum_{k=1}^{m} N_k x_i u_i(k) + \sum_{j=1}^{n} a_{ij}^M F x_j, \qquad x_i(0) \in \mathbb{R}^{n_x}, \qquad \forall i \in \{1, \cdots, n\},$$

$$u_i \in \mathcal{V}, \qquad \mathcal{V} = \{v^1, \cdots, v^N\} \subset \mathbb{R}^{n_u},$$

  $\mathcal{V}$ is non-empty and contains 0 in its interior.

- Investigate more complex Lyapunov functions

Conclusion

- Switching control for networks of switched affine systems
- Decentralized controllers
- Scalable design conditions

Thank you!

References

- Bolzern, Paolo and William Spinelli. "Quadratic stabilization of a switched affine system about a nonequilibrium point". In: *Proceedings of the 2004 American Control Conference*. Vol. 5. IEEE. 2004, pp. 3890–3895.
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