Integral Active Disturbance Rejection Control: Application to Load Frequency Control Problem

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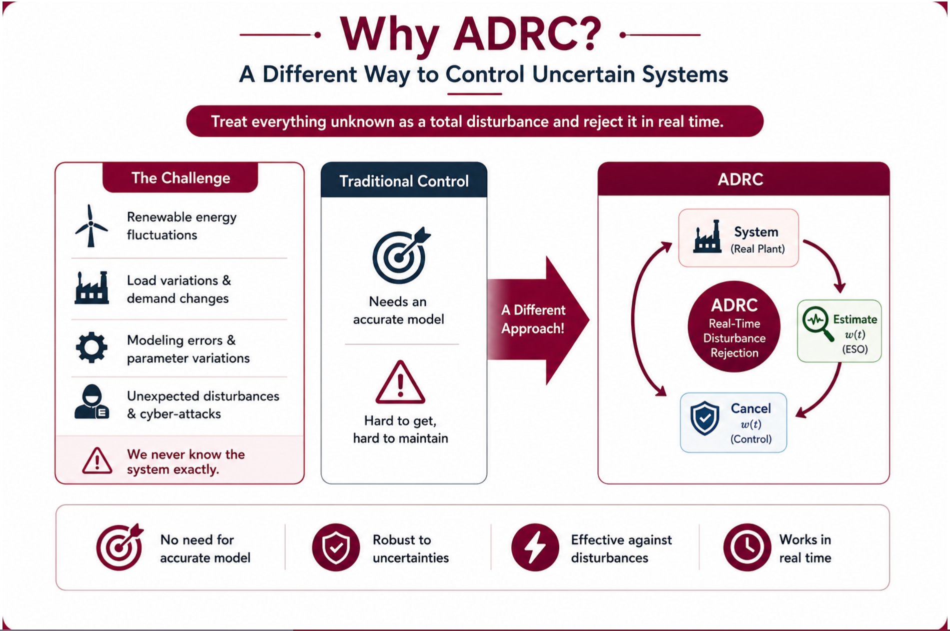

Why ADRC?
A Different Way to Control Uncertain Systems
Treat everything unknown as a total disturbance and reject it in real time.
The Challenge
Renewable energy fluctuations
Load variations & demand changes
Modeling errors & parameter variations
Unexpected disturbances & cyber-attacks
We never know the system exactly.
Traditional Control
Needs an accurate model
Hard to get, hard to maintain
A Different Approach!
ADRC
System (Real Plant)
ADRC Real-Time Disturbance Rejection
Estimate $w(t)$ (ESO)
Cancel $w(t)$ (Control)
No need for accurate model
Robust to uncertainties
Effective against disturbances
Works in real time

Contents

Active Disturbance Rejection Control (ADRC)

Consider an n-th order system affected by unknown dynamics and disturbances:

$$y^{(n)}(t) = b\,u(t) + f\big(y(t), \dot{y}(t), \ldots, y^{(n-1)}(t), u(t), d(t)\big), \tag{1}$$

We can rewrite the above system as follows

$$y^{(n)}(t) = b_0 u(t) + \omega(t), \quad \omega(t) := (b - b_0)u(t) + f(\cdot). \tag{2}$$

Objective

The goal is to make $y(t)$ track $r(t)$ using $u(t)$, without requiring explicit knowledge of the system dynamics, and ω(t) is treated as an additional state which is estimated and cancelled.

Thus, the system is transformed from a model based problem into a **disturbance rejection problem**

ADRC- Continued

To design the ADRC, we write the system in compact state-space form as follows

$$\dot{x}(t) = Ax(t) + Bu(t) + E\dot{\omega}(t), \tag{3}$$

where

$$x(t) = \begin{bmatrix} y & \dot{y} & \cdots & y^{(n-1)} & \omega \end{bmatrix}^T,$$

A Luenberger-type ESO is employed to estimate the states and total disturbance:

$$\dot{\hat{z}}(t) = (A - L_oC)\hat{z}(t) + Bu(t) + L_oy(t), \quad \hat{y}(t) = C\hat{z}(t), \tag{4}$$

ADRC-Continued

Extended State Observer (ESO):

$$\dot{\hat{z}}_1 = \hat{z}_2 + \beta_1(y - \hat{z}_1), \tag{5}$$

$$\vdots$$

$$\dot{\hat{z}}_n = \hat{z}_{n+1} + \beta_n(y - \hat{z}_1) + b_0 u, \tag{6}$$

$$\dot{\hat{z}}_{n+1} = \beta_{n+1}(y - \hat{z}_1). \tag{7}$$

Let $r(t)$ be the reference signal and using the disturbance estimate $\hat{z}_{n+1}(t)$, the control law is defined as:

Control Law

$$u(t) = \frac{1}{b_0}\left[\sum_{i=1}^{n} K_i\left(r^{(i-1)}(t) - \hat{z}_i(t)\right) - \hat{z}_{n+1}(t)\right]. \tag{8}$$

Active Disturbance Rejection Control - Continued

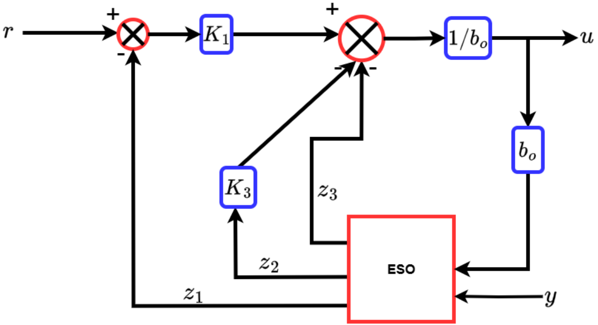


Figure 1: Block Diagram of ADRC

- **Drawbacks of ADRC :**
  - Steady-state offset may remain despite disturbance-rejection mechanisms because $d(t)$ is cancelled dynamically.
  - Performance may degrade when disturbances are low-frequency or persistent, or when dynamics are slow.

Integral Active Disturbance Rejection Control (I-ADRC)

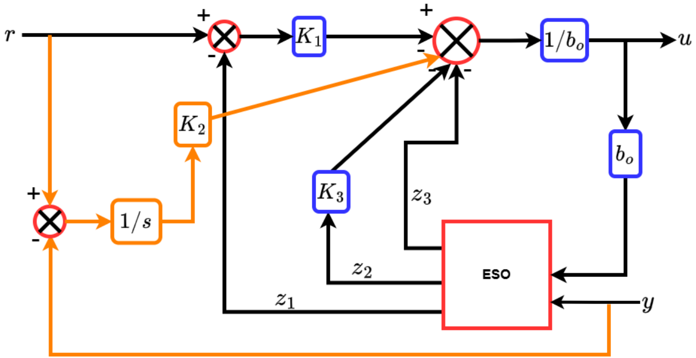


Figure 2: Block Diagram of I-ADRC

Concept:
I-ADRC extends ADRC by adding an **integral term** to reduce steady-state error while maintaining robustness against disturbances and uncertainties.

IADRC Continued

$$e(t) = r(t) - y(t), \qquad \alpha(t) = \int_0^t e(\tau)\, d\tau$$

Control Law (with Integral Action)

$$u(t) = \frac{1}{b_0}\left[\sum_{i=1}^{n} K_i\left(r^{(i-1)}(t) - \hat{z}_i(t)\right) + K_I\alpha(t) - \hat{z}_{n+1}(t)\right]. \tag{9}$$

Key Advantages:

- Integrates ADRC's real-time disturbance rejection with integral error correction.
- Can achieve zero steady-state error.
- Enhances robustness under model uncertainties and cyber-attacks.

Points

- How to provide stability guarantees?
- How to tune the parameters?

Stability Analysis

We redefine the dynamics in terms of the tracking error and observer error as follows:

$$\begin{cases} \dot{\boldsymbol{\varepsilon}}(t) = A_c \boldsymbol{\varepsilon}(t) + B_c \tilde{\boldsymbol{z}}(t) + G(\ddot{r}(t) - \dot{\omega}(t)), \\ \dot{\tilde{\boldsymbol{z}}}(t) = (A_e - L_o C_e)\tilde{\boldsymbol{z}}(t) + E\,\dot{\omega}(t), \end{cases} \tag{10}$$

where

$$\boldsymbol{\varepsilon}(t) = \begin{bmatrix} e(t) \\ \dot{e}(t) \\ \alpha(t) \end{bmatrix} = \begin{bmatrix} r(t) - y(t) \\ \dot{r}(t) - \dot{y}(t) \\ \displaystyle\int_0^t e(\tau)\, d\tau \end{bmatrix}, \quad \tilde{\boldsymbol{z}}(t) = \boldsymbol{x}(t) - \hat{\boldsymbol{z}}(t),$$

denote the tracking and estimation error vectors, respectively. Introducing the augmented state as $\bar{x}(t) := [\, \varepsilon(t),\, \tilde{\boldsymbol{z}}(t)\,]^\top$, we get

$$\dot{\bar{x}}(t) \;=\; \begin{bmatrix} A_c & B_c \\ 0 & A_e - LC_e \end{bmatrix} \bar{x}(t) \;+\; \begin{bmatrix} -G & G \\ E & 0 \end{bmatrix} \begin{bmatrix} \dot{\omega}(t) \\ \ddot{\tilde{r}}(t) \end{bmatrix} \;:=\; A_{CL}\bar{x}(t) \;+\; \bar{E}g(t). \tag{11}$$

Stability Analysis- Continued

Theorem

If there exist $0 < P = P^\top \in \mathbb{R}^{n\times n}$, $0 < Q = Q^\top \in \mathbb{R}^{n\times n}$, $0 < \Gamma = \Gamma^\top \in \mathbb{R}^{p\times p}$ *such that the LMIs*

$$\begin{bmatrix} A_{CL}^\top P + P A_{CL} + Q & P\bar{E} \\ \bar{E}^\top P & -\Gamma \end{bmatrix} \leqslant 0$$

are satisfied. Then the errors $\epsilon(t)$ *and* $\bar{z}(t)$ *are Input-to-State stable with respect to disturbances* $\dot{\omega}(t)$ *and* $\ddot{\bar{r}}(t)$.

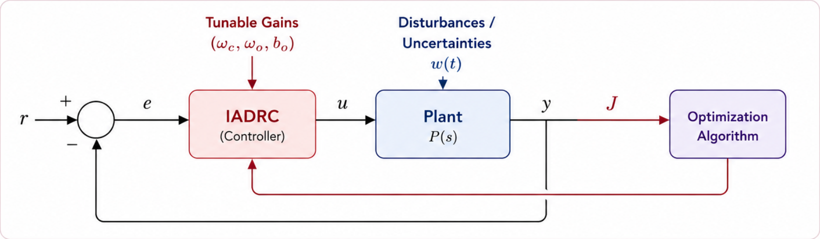
The Proof can be found here

Altaf A, Khalin A, Bourdais R. Integral active disturbance rejection control for microgrid load frequency regulation under cyberattacks. ***Control Engineering Practice***. 2026 Jul 1;172:106921.

IADRC optimization problem

IADRC Optimization Problem

Optimize IADRC parameters to achieve the best closed-loop performance under **disturbances and uncertainties.**



Objective Function

$$\min_{\omega_c, \omega_o, b_o} \quad J(\omega_c, \omega_o, b_o) = \int_0^T |J_1| dt + \int_0^T |J_2| dt$$

subject to

$$\begin{aligned} \omega_{c,\min} &\leq \omega_c \leq \omega_{c,\max} \\ \omega_{o,\min} &\leq \omega_o \leq \omega_{o,\max} \\ b_{o,\min} &\leq b_o \leq b_{o,\max} \end{aligned}$$

Two Scenarios (J_1 and J_2)

Scenario 1 (J_1)

- Disturbance / Attack Set 1
- Operating Condition 1
- Cost Function: J_1

Scenario 2 (J_2)

- Disturbance / Attack Set 2
- Operating Condition 2
- Cost Function: J_2

Application- Load Frequency Control problem

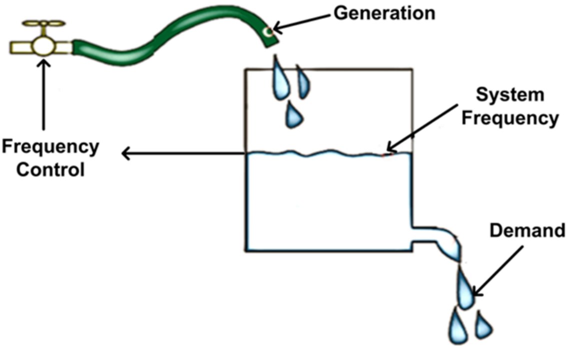


Figure 3: General Analogy of Load Frequency Control.

LFC maintains system frequency by adjusting generator outputs in response to load changes.

- **High Renewable Penetration**
- **Cyber-Physical Threats and Communication delays**
- **Model Uncertainty**

Application- Load Frequency Control problem

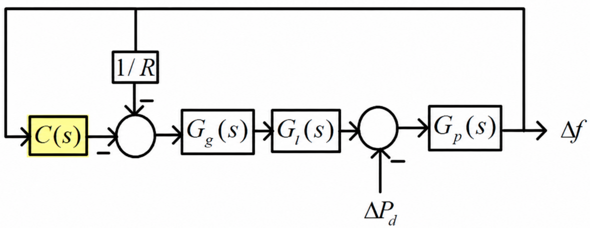


Figure 4: Block Diagram of LFC of microgrid

The aim is to contrive a control law

$$u(s) = C(s)\,\Delta f(s),$$

such that

$$\lim_{t\to\infty} \Delta f(t) = \lim_{s\to 0} s\,\Delta f(s) = 0,\ \forall\,\Delta P_d,$$

where $C(s)$ is the controller function, ΔP_d is the load disturbance (p.u. MW) which is a bounded function ($|\Delta P_d| \leqslant \Delta P_{d\,\max}$), Δf is the frequency deviation (Hz), and $u(s)$ is the control input.

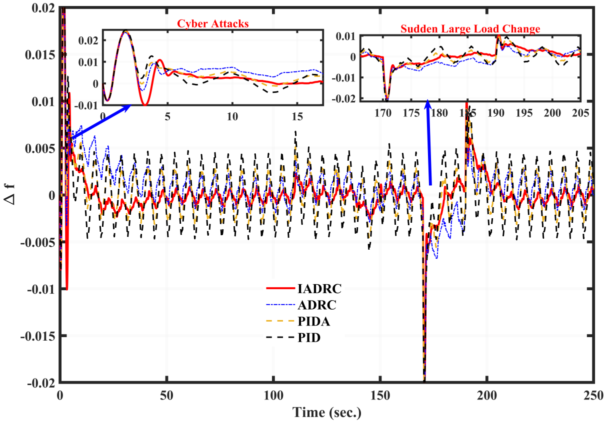


Figure 5: Δf responses when -50% PU in T_t and T_g and -70% in H and D of microgrid

Stability and Error Evolution

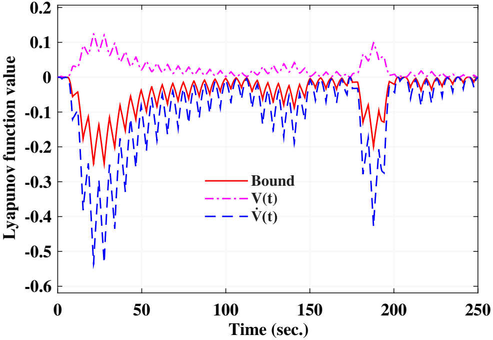
- To proceed, we need to transform the dynamics into the required form for IADRC as in (2).
- Recall that we measure the frequency deviation, which can be represented as our output $y = \Delta f$.
- Converting the swing equation into state-space and taking the second derivative, we get

$$\ddot{y} = \Delta\ddot{f} = \frac{1}{2H}\Big(\Delta\dot{P}_m + \Delta\dot{P}_{wind} + \Delta\dot{P}_{pv} + \Delta\dot{P}_{bes} + \Delta\dot{P}_{fes} - D\Delta\dot{P}_{load}\Big) := \omega + b_0 u. \tag{12}$$

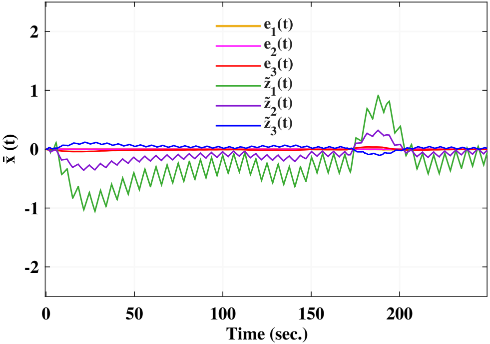
From which, we define ω for LFC system as follows

$$\omega = \frac{1}{2H}\Big(\Delta\dot{P}_m + \Delta\dot{P}_{wind} + \Delta\dot{P}_{pv} + \Delta\dot{P}_{bes} + \Delta\dot{P}_{fes} - D\Delta\dot{P}_{load}\Big) - b_0 u. \tag{13}$$

Stability and Error Evolution - Continued.



(a) Lyapunov $V(t)$, its derivative $\dot{V}(t)$ and bound evolution.



(b) Tracking and observer error evolution $\bar{x}(t)$.

Figure 6: Stability and Error Evolution.

Conclusion

- In this work, we proposed an IADRC design for the single-area microgrid LFC problem, enhancing frequency regulation and resilience to bounded cyber attacks, with stability guaranteed through input-to-state stability analysis and parameters tuned via co-simulation (metaheuristics).

This work has been published in Control Engineering Practice and will be presented at 23rd IFAC World Congress.

Thank you!