Réduction des ondulations de couple pour machines synchrones à réluctance par schéma d'annulation adaptatif

SAGIP 2026 — CSE: Commande des Systèmes Électriques (TD 09)

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Outline

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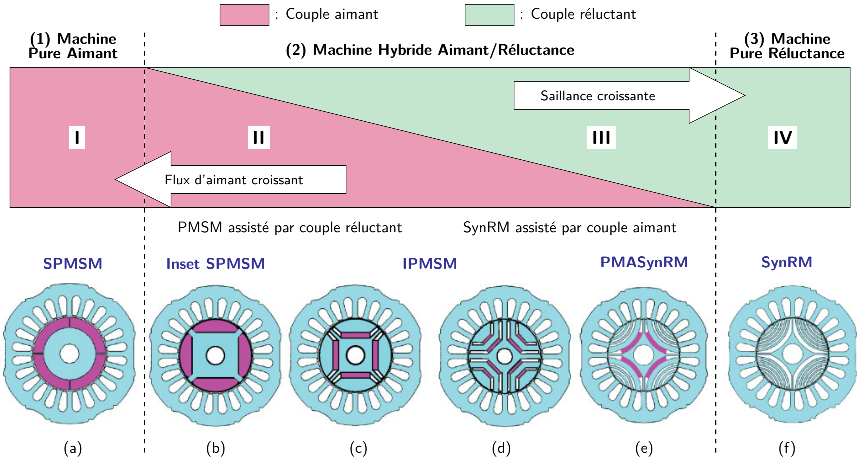
Synchronous Machines Classification



Adapted from: [1]

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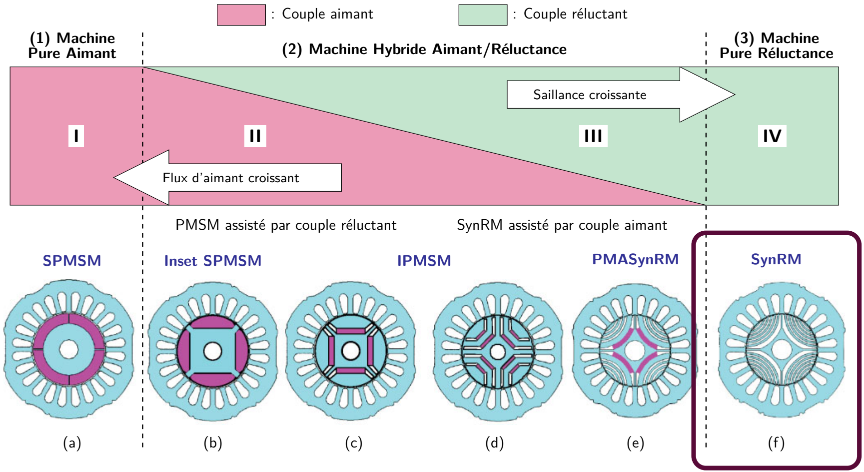

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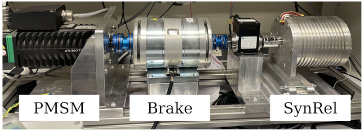
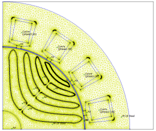
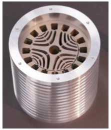
The Magnetic Complexity of SynRel

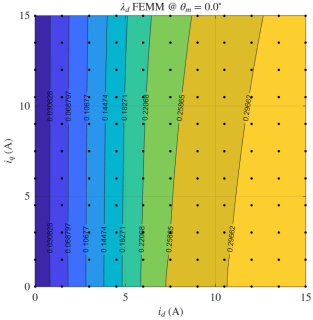


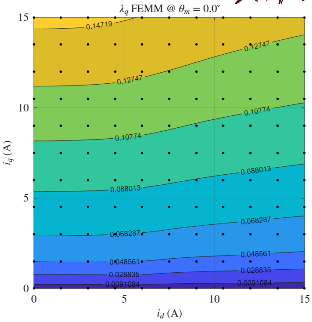
SynRel Test Bench





FEMM Mesh & Prototype Motor





$\lambda_d(i_d, i_q, \theta_m)$ and $\lambda_q(i_d, i_q, \theta_m)$ — $\theta_m \in [0, 180]^\circ$ — $L(\theta) = \nabla_i \lambda$

- **FEMM Simulation** $\Rightarrow$ fluxes $\lambda_{d,q}(i_d, i_q, \theta_m)$ and torque $\tau_m(i_d, i_q, \theta_m)$.
- **θ_m-dependency** $\Rightarrow$ torque harmonics (saliency + slots).
- **Nonlinear system (saturation)** $\Rightarrow$ nonlinear torque harmonics.
- **Custom built 2-phase SynRel** [2], [3] enables injection of any current harmonics.

Mathematical Model in dq Frame

Current Dynamics Derivation

Flux Derivative (accounting for spatial harmonics):

$$\frac{d\lambda_{dq}}{dt} = \frac{\partial \lambda_{dq}}{\partial i_{dq}} \frac{di_{dq}}{dt} + \frac{\partial \lambda_{dq}}{\partial \theta_m} \omega_m$$

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$$\mathcal{L}_{dq} \frac{di_{dq}}{dt} = v_{dq} - R_s i_{dq} - \frac{\partial \lambda_{dq}}{\partial \theta_m} \omega_m + p\omega_m J \lambda_{dq}$$

Incremental Inductance

Jacobian of the non-linear flux map:

$$\mathcal{L}_{dq}(i_d, i_q, \theta_m) = \frac{\partial \lambda_{dq}}{\partial i_{dq}} = \begin{bmatrix} \frac{\partial \lambda_d}{\partial i_d} & \frac{\partial \lambda_d}{\partial i_q} \\ \frac{\partial \lambda_q}{\partial i_d} & \frac{\partial \lambda_q}{\partial i_q} \end{bmatrix}$$

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EM torque:

$$\tau_{em} = \frac{\partial W'(i_d, i_q, \theta_m)}{\partial \theta_m} + p \lambda_{dq}^{\top} J i_{dq}$$

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Magnetic Coenergy

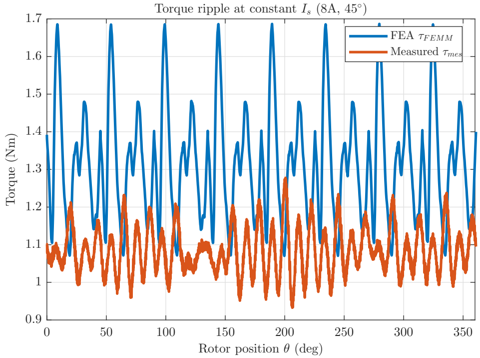
The energy stored in the magnetic field:

$$W'(i_d, i_q, \theta_m) = \int_0^{i_d} \lambda_d(i'_d, 0, \theta_m) di'_d + \int_0^{i_q} \lambda_q(i_d, i'_q, \theta_m) di'_q$$

Context & Objectives

Systemic Bottlenecks

- **Parametric uncertainty:** R_s drifts, inverter distortions *[4]*
- **Sensing limits:** Torque sensors lack bandwidth
- **Non-linearities:** Saturation and spatial harmonics generate periodic terms
- Discrepancy between FEA and torque sensor (non modeled losses)



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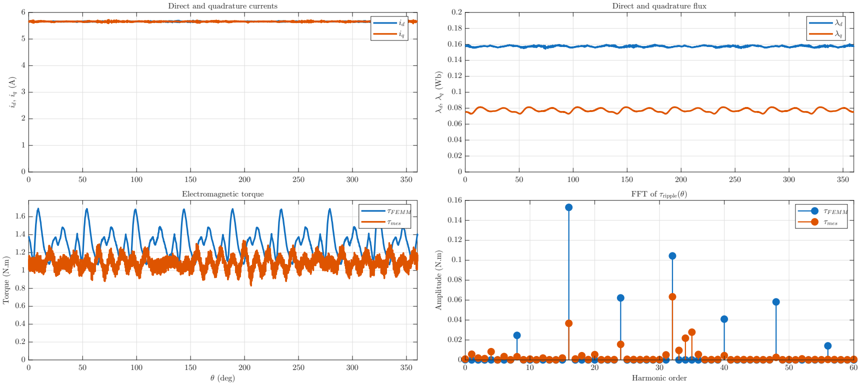
Proposed Strategy

1. **Pragmatic Baseline**
   - Standard PI current loops using average $\bar{\mathcal{L}}_{dq}$
2. **Adaptive Rejection (Our Goal)**
   - Treat non-linearities as **periodic disturbances**
   - Reject torque ripples via **harmonic *dq* injection**
   - Use **FEA mapping** as a virtual trajectory sensor

Outline

FEA vs Measured Torque: Matching Harmonic Orders

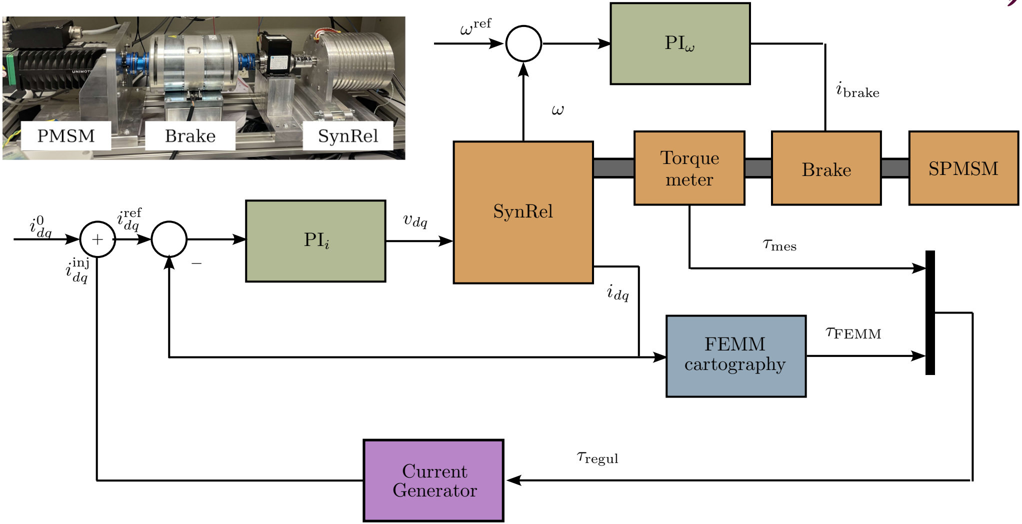
Virtual Sensor: Instead of measuring mechanically, we estimate the torque $\tau_{\mathrm{FEMM}}(i_d, i_q, \theta_m)$ using the FEMM analysis to capture torque ripple harmonics.



FEA and measured torque for $i_{\mathrm{dq}} = I_s[\cos(\beta), \sin(\beta)]^\top$ *with* $I_s = 8\,\mathrm{A}$ *and* $\beta = 45^\circ$*.*

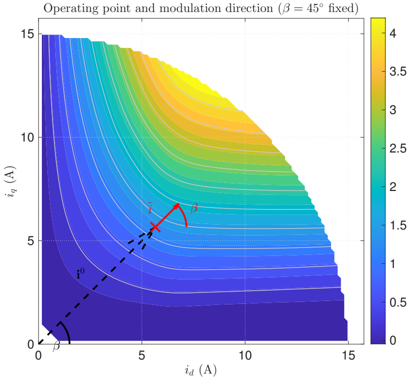
$\Rightarrow$ **Same harmonic orders, mismatched amplitudes.**

Experimental Setup



Local LTI Approximation

To apply standard adaptive control, we verify that the system behaves approximately as a linear gain K_τ near the operating point (I_s^0, β^0).



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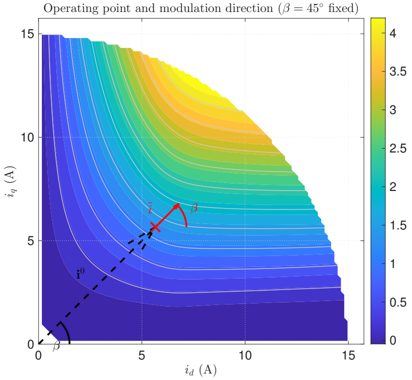
Torque Sensitivity K_τ (FEA based)

$$K_\tau(\theta) = \left.\frac{\partial \tau}{\partial I_s}\right|_{I_s^0, \beta}$$

The DC component of K_τ dominates.

$\Rightarrow$ The system P acts locally as a Linear Time-Invariant (LTI) system:

$$\tilde{\tau} \approx K_\tau \tilde{\imath}^{\mathrm{inj}}$$



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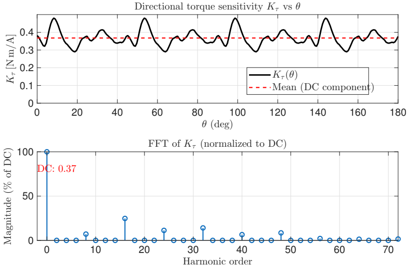

Bodson Adaptive Scheme

Modulation Law

$$\dot{z} = -g \begin{bmatrix} \cos(\varphi) & \sin(\varphi) \\ -\sin(\varphi) & \cos(\varphi) \end{bmatrix}^{-1} \begin{bmatrix} \cos(k\theta) \\ \sin(k\theta) \end{bmatrix} \tau_{hf}(t)$$

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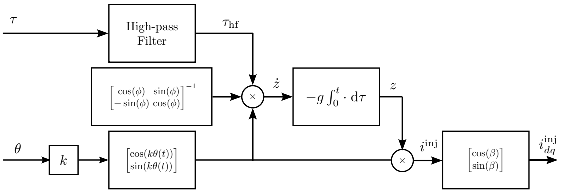
Current Injection

$$i_{\mathrm{dq}}^{\mathrm{ref}}(t) = i_{\mathrm{dq}}^{0} + \begin{bmatrix} \cos(\beta) \\ \sin(\beta) \end{bmatrix} z(t)^{\top} \begin{bmatrix} \cos(k\theta) \\ \sin(k\theta) \end{bmatrix}$$



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Multi-harmonic: one block per order k_i

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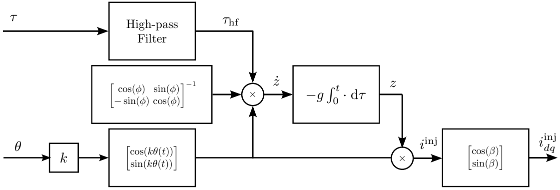

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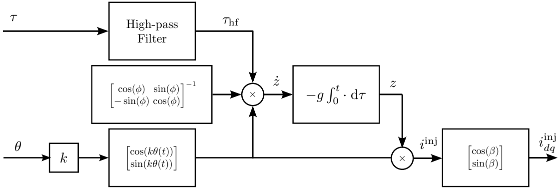
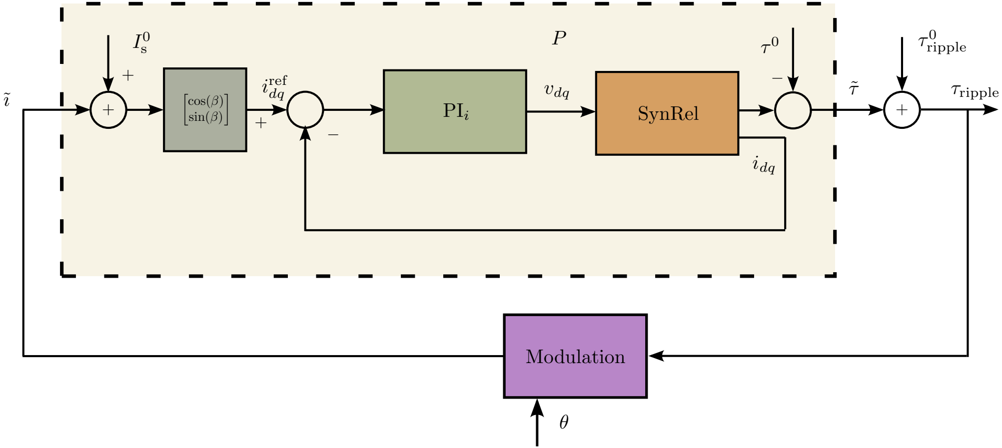


- Equivalent to resonant controller or repetitive control strateg [5]
- Doesn't require precise plant model
- z is the amplitude estimator, φ is the phase offset
- g is a "small" gain (tuning parameter)

The Control Architecture



The plant P is the closed loop $i_{ref} \to \tau$.

Discussing the Strategy

Stability Guarantee:

Stable as long as:

- g is small enough (but not too small)
- $\varphi = \angle P(\mathrm{j}k\omega^0) \pm 90^\circ$ (SPR condition)

where P is the LTI approx. of the dynamics at the given point.

$\Rightarrow$ Facilitates empirical tuning of the controller.

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$\Rightarrow$ Facilitates empirical tuning of the controller.

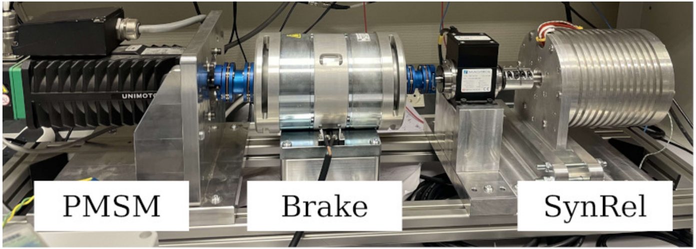
- $\angle P(\mathrm{j}k\omega^0)$ easily measurable from experimental current injection
- $g \approx 0.1$ as a starter

Outline

Experimental Setup

Operating Point:

- Speed: $200\,\mathrm{rpm}$
- Current I_s: $8\,\mathrm{A}$
- Angle β: $45°$
- Harmonics targeted: H_{16}, H_{32}

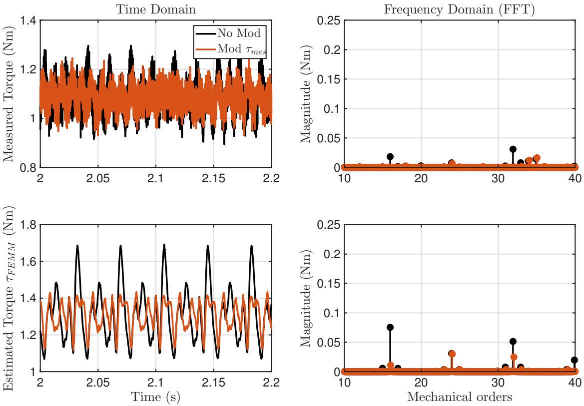


Results: Feedback on τ_{mes} (Physical Sensor)

Baseline Performance: Using the physical torque sensor provides a baseline for maximal harmonic rejection.

Attenuation:

- H_{16}: 99.5 %
- H_{32}: 99.8 %



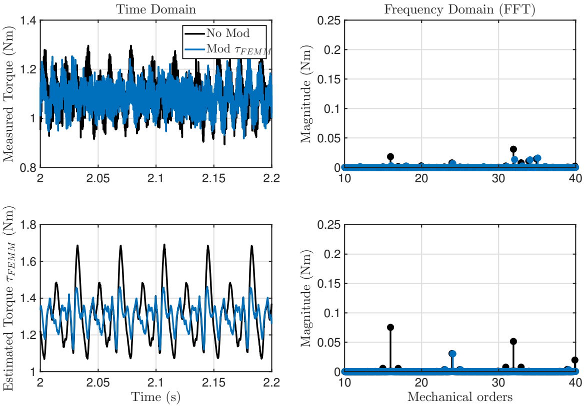
Results: Feedback on τ_{FEMM} (Virtual Sensor)

Sensorless Performance

Replacing the mechanical sensor with the FEMM virtual sensor maintains significant harmonic attenuation.

Attenuation in real measured torque:

- H_{16}: **84.6 %**
- H_{32}: **56.5 %**



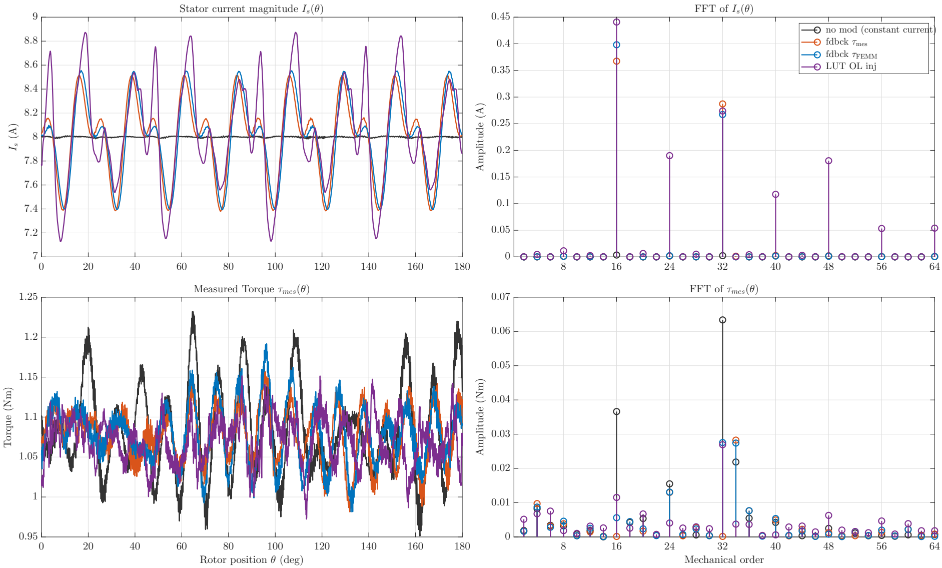
Quantitative Performance Summary

Table: Cross-Analysis: Attenuation vs Feedback Source

	Attenuation (%)				Injected Current i_{dq}^{inj}			
	Order 16		Order 32		Order 16		Order 32	
Feedback Source	τ_{mes}	τ_{FEMM}	τ_{mes}	τ_{FEMM}	Amp. (A)	Ang. (°)	Amp. (A)	Ang. (°)
τ_{mes}	**99.5**	85.8	**99.8**	51.5	0.37	60.9	0.29	-120.2
τ_{FEMM}	84.6	**99.8**	56.5	**99.9**	0.40	51.7	0.27	-152.0

- **Sensor-based loop**: Maximal attenuation of torque ripples, but requires a mechanical sensor.
- **Sensorless (FEMM)**: Significant attenuation (**84.6 %** at H_{16}) achieved without mechanical feedback.

Comparison: Adaptive Modulation vs. Optimal LUT Rejection



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Conclusion

- **Robust Cancellation:** Bodson adaptive control effectively reduces torque ripple without relying on a perfect feedforward model.
- **No Mechanical Sensor:** The FEA torque estimator (τ_{FEMM}) can completely replace the mechanical torque sensor in the feedback loop.
- **Experimental Proof:** $> 84\,\%$ reduction on 16th harmonic validated on a real SynRel bench.

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Next Steps & Perspectives

- Remove the reliance on the FEA pre-computed map by developing a **real-time flux observer** to reconstruct torque directly from electrical measurements.
- Investigate **Linear Time-Periodic (LTP)** and **Harmonic State Space (HSS)** modeling of the machine for advanced controller synthesis.

Thank you for your attention

Any questions?

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[3] R. Delpoux, T. Huguet, F. B. Argomedo, L. Queval, J.-Y. Gauthier, and Z. Kader, “Finite element dq-model for MTPA flux control of Synchronous Reluctance Motor (SynRM),” in *2023 IEEE 32nd International Symposium on Industrial Electronics (ISIE)*, IEEE, Jun. 2023, pp. 1–6. DOI: 10.1109/isie51358.2023.10227915

[4] S. Alhaj Hassan, R. Delpoux, V. Léchappé, T. Huguet, and X. Brun, “Impact of inverter nonlinear effects on observer-based flux estimation for synrel machines,” in *2026 IEEE International Conference on Industrial Technology (ICIT)*, 2026. DOI: 10.1109/ICIT64854.2026.11491203

[5] M. Bodson and S. C. Douglas, “Adaptive algorithms for the rejection of sinusoidal disturbances with unknown frequency,” *IFAC Proceedings Volumes*, vol. 29, no. 1, pp. 5168–5173, Jun. 1996, ISSN: 14746670. DOI: 10.1016/S1474-6670(17)58501-8

Appendix

Model in *ab* Frame

Physical State Model

Flux Dynamics:

$$\frac{d\lambda_{ab}}{dt} = -R_s i_{ab} + v_{ab}$$

Electromagnetic Torque:

$$\tau_{em} = \frac{\partial W'(i_a, i_b, \theta_m)}{\partial \theta_m}$$

Magnetic Coenergy

The energy stored in the magnetic field:

$$W'(i_a, i_b, \theta_m) = \int_0^{i_a} \lambda_a(i_a', 0, \theta_m) di_a' + \int_0^{i_b} \lambda_b(i_a, i_b', \theta_m) di_b'$$

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Practical Challenges:

- Resistance R_s is **poorly known** and varies.
- Voltages v_{ab} are heavily distorted by **inverter non-linearities** (dead-times, voltage drops).

[4]

Why inject along the β axis?

1. Directional Sensitivity

We seek the injection direction β_{inj} that maximizes torque ripple cancellation per ampere injected.
This corresponds to the torque gradient:

$$\beta_{\mathrm{opt}} = \arg\left(\nabla_{i_{\mathrm{dq}}} \tau(i^0_{\mathrm{dq}}, \theta)\right)$$

2. The MTPA Property

The nominal operating point β^0 is typically chosen on the **Maximum Torque Per Ampere (MTPA)** trajectory.
By definition, along MTPA, the torque gradient is collinear with the current vector:

$$\nabla_{i_{\mathrm{dq}}} \tau \parallel i^0_{\mathrm{dq}}$$

Conclusion

If the system operates at MTPA ($\beta^0 = \beta_{\mathrm{mtpa}}$), then $\beta_{\mathrm{opt}} = \beta^0$.

Injecting along the nominal β trajectory is mathematically optimal!

Non-linear torque map & MTPA